ENERGY TRANSFER FROM AN INDUCTIVE STORAGE BY MEANS OF AN EXPLOSIVE CURRENT DISCONNECT

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Energy transfer from an inductive storage is considered for two types of systems: a disconnect with an intrinsic parasitic inductance for an inductive load and a purely resistive disconnect for a resistive load. Solutions are obtained for the voltage, power, and energy transferred to the load. The dependence of the efficiency of the device on its parameters is established.

1. Transfer of Energy from an Inductive Storage to an Inductive Load Including the Parasitic Inductance of the Current Disconnect. The electrical circuit of Fig. 1 corresponds to the differential equation

$$L_e I_d' + R I_d = 0,$$

where $L_e = L_d + LL_L/(L + L_L)$ is the equivalent inductance of the circuit (with respect to the disconnect). Considering the initial conditions ($t = 0$, $I_L = 0$, $I = I_d = I_0$), we then find

$$I_d = I_0 \exp - \frac{R(t)}{L_e} t;$$

$$Q_d = Q_0 \left(\frac{L_d}{L} + \frac{L_L}{L + L_L}\right)\left[1 - \left(\frac{I_d}{I_0}\right)^2\right];$$

$$Q_L = Q_0 \frac{LL_L}{(L + L_L)^2}\left(1 - \frac{I_d}{I_0}\right)^2.$$

These relations are similar to the equations obtained in [1] for a purely resistive disconnect; the difference is that in the present case L_e has the additional term L_d and Q_d, the additional term $Q_0 L_d/L$. The latter quantity is the magnetic field energy stored in the inductance of the disconnect at the initial time. Thus, with a parasitic inductance present in the disconnect, the same fraction of the energy is transferred from the storage into the load as in the case of a noninductive disconnect, although the disconnect process takes place in a different way. In particular, the energy absorbed by the disconnect is different; the magnetic energy stored in the parasitic inductance is completely absorbed, in addition to a certain fraction of the stored energy. Because of this, the total efficiency of the system is lowered. The nature of the process of energy transfer is primarily determined by the value of the parameter $A = L_e I_0^2/2m_0 q$, where m_0 and q are the initial mass of the disconnect and the specific energy of the electrical explosion. All the expressions obtained in [1] for the dimensionless quantities characterizing the energy-transfer process remain valid. It should be noted that although the voltage U applied to the load and disconnect has an inductive component LI_d', in this case, in addition to the resistive component RI_d, the expression for the dimensionless quantity ($u = U/U_0$) remains the same as that when $L_d = 0$, and only the initial value

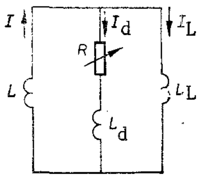

Fig. 1

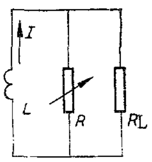

Fig. 2

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$$U_0 = U(0) = (1 - L_d/L_e)\, R_b I_0$$

is changed.

2. Energy Transfer from an Inductive Storage to a Resistive Load. First of all, we point out the basic difference between energy transfer into an inductive load and into a purely resistive load. In the first case, as is clear from the equations for Q_d and Q_L, the value of the current in the disconnect (or load) branch completely determines the value of the energy transferred to the load and that left in the storage. In particular, the energy transferred to the load for complete cutoff is determined only by the initial conditions and does not depend on the manner in which the cutoff is accomplished in time. This energy is not changed even if the method of cutoff is itself changed, for example, by replacing electrical explosion of the current disconnect by a purely mechanical rupture. A different situation arises in the transfer of energy to a purely resistive load. In this case, Q_L is not determined by the value of the current I_L itself but by the entire integral $\int_0^t R_L I_L^2(t)\, dt$, i.e., it depends on the nature of the disconnect process. Therefore, it is impossible to determine the energy transferred to the load for total cutoff merely through the initial conditions; it is certainly necessary to take into account the nature of the disconnect process. For example, if instantaneous cutoff is accomplished, all the stored energy is transferred to the load. In fact, no matter how rapidly the disconnect process occurs, the power in the disconnect is limited [it cannot exceed the value $I_0^2 R_L^2 / R(t)$], and therefore $Q_d \to 0$ when $t \to 0$. We now turn to a study of the electrical circuit in Fig. 2 described by the system of differential equations

$$LU'\left(\frac{1}{R} + \frac{1}{R_L}\right) - LU\frac{R'}{R^2} + U = 0, \tag{2.1}$$

$$R' = aRU^2, \tag{2.2}$$

where $a = (m_0 q R_b)^{-1}$.

Taking for the electrical explosion of a wire the model based on a surface vaporization wave [2], we obtain

$$R = R_b(1 - Q_d/m_0 q)^{-1},$$

where R_b is the resistance of the disconnect at the boiling point ($R = R_b$ at $t = 0$, $I = I_0$, $U = U_0 = I_0 R_b R_L/(R_b + R_L)$. Eliminating U from Eqs. (2.1) and (2.2) and integrating, we obtain

$$R' = \frac{2R^3}{(R + R_L)^2}\left[\frac{R_L^2}{LR} - \frac{R_L}{L}\ln\frac{R}{R_b} + \frac{aR_L^2 I_0^2}{2} - \frac{R_L^2}{LR_b}\right].$$

We introduce the dimensionless quantities

$$r = \frac{R}{R_b},\quad r_L = \frac{R_L}{R_b},\quad \tau = \frac{t}{L/R_b},\quad A = \frac{LI_0^2}{2m_0 q},\quad i = \frac{I}{I_0},\quad u = \frac{U}{U_0}.$$

We then obtain

$$r' = \frac{dr}{d\tau} = \frac{2r^3 r_L}{(r + r_L)^2}\left[\frac{r_L}{r} - \ln r + (A - 1)\, r_L\right]. \tag{2.3}$$

It is then clear that r tends asymptotically to its limit r_1, which is determined by the condition $r' = 0$,

$$\ln r_1 = r_L\left(A - 1 + \frac{1}{r_1}\right). \tag{2.4}$$

Thus, an electrical explosion, in the proper sense of the word, does not occur; the resistance R is always finite for any finite R_L, i.e., the mass of the disconnect cannot be completely vaporized. The limiting value r_1 depends on the parameters A and r_L with the nature of this dependence being determined by the value of A. For $A < 1$ (low-energy mode), Eq. (2.4) indicates that

$$r_1 < \frac{1}{1 - A}$$

for any r_L. If $r_L \to \infty$, $r_1 \to (1 - A)^{-1}$, and consequently, it is impossible in this mode to obtain a significant increase in the resistance of the disconnect and efficient transfer of energy to the load. For $A \leq 0.5$,

$$\ln r_1 \cong r_1 - 1, \qquad \frac{1}{r_1} \cong 1 - (r_1 - 1).$$

We then obtain from Eq. (2.4)

$$r_1 \cong 1 + A\frac{r_L}{1+r_L}. \tag{2.5}$$

For A > 1 (high-energy mode), it is clear from Eq. (2.4) that

$$\ln r_1 > r_L(A-1)$$

and this means

$$\frac{1}{r_1} < e^{-r_L(A-1)}, \quad \ln r_1 < r_L\left[(A-1) + e^{-r_L(A-1)}\right].$$

Hence, we have for an estimate of r_1

$$r_L(A-1) < \ln r_1 < r_L\left[(A-1) + e^{-r_L(A-1)}\right].$$

For $A \geq 2$ and $r_L \geq 1$,

$$r_1 \cong e^{r_L(A-1)},$$

i.e., the resistance of the disconnect in this mode, although also finite, may be large for sufficiently large r_L. If, indeed, $A > 1$ but r_L is small so that $Ar_L < 1$, we once again obtain Eq. (2.5) for r_1 from Eq. (2.4).

We turn to a calculation of the voltage on the load. The greater the power or energy required to be transferred to the load in a limited time, the steeper the rise of the voltage pulse to a maximum must be.

However, analysis shows that modes are possible where the pulse has the form of a monotonically damped curve. In fact, at the time of maximum voltage τ_M, $u = u_M$, $u' = 0$, $r = r_2$, and $r' = r_2^2$ from Eq. (2.1). Using Eq. (2.3), we obtain for r_2 the transcendental equation

$$1 - 2\frac{r_2}{r_L}(\ln r_2 + 1) + 2(A-1)r_2 - \left(\frac{r_2}{r_L}\right)^2 = 0. \tag{2.6}$$

It does not always have a solution (i.e., the presence of a maximum in the voltage curve is not obligatory). We set $r_2 = 1$ in Eq. (2.6) and find the corresponding value of $r_L = r_{cr}$ (where r_{cr} is the critical value of the load resistance). It is clear that Eq. (2.6) does not have a solution for small r_L, since r_2 cannot be less than one. We find for r_{cr}

$$r_{cr} = \frac{1}{\sqrt{2A} - 1} \quad \text{for} \quad A > \frac{1}{2},$$

and there is no solution for $A \leq 1/2$. The dependence of r_{cr} on A is shown in Fig. 3 in the form of a curve above which lies the region of pulses with maxima and below which pulses are monotonically damped. The dependence of r_2 on A and r_L is most conveniently obtained by a solution of Eq. (2.6) with respect to r_L,

$$r_L = \frac{r_2(\ln r_2 + 1)}{1 + 2(A-1)r_2}\left[1 + \sqrt{1 + \frac{1 + 2(A-1)r_2}{(\ln r_2 + 1)^2}}\right]$$

and by plotting the corresponding curves (Fig. 4a). The amplitude of the voltage pulse is found through the substitution $r' = r_2^2$ in Eq. (2.2)

$$u_M = \left(1 + \frac{1}{r_L}\right)\sqrt{\frac{r_2}{2A}}.$$

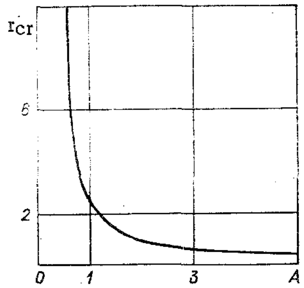

Fig. 3

For $1/2 < A < 1$ and $r_L \to \infty$, Eq. (2.6) leads to

$$r_2 \to \frac{1}{2(1-A)},$$

i.e., in this case the voltage reaches a maximum when the resistance of the disconnect becomes equal to half the limiting value. The amplitude of the voltage pulse then tends to the limit

$$u_M \xrightarrow[r_L \to \infty]{} \frac{1}{2\sqrt{A(1-A)}}, \quad \frac{1}{2} < A < 1.$$

It is clear that it is impossible to obtain high voltages for $A < 1$ because $u_M \to 1.67$ even for $A = 0.9$ and $r_L \to \infty$. For $A \geq 1$, r_2 and u_M in theory increase without limit as r_L increases. However, it is known from experi-

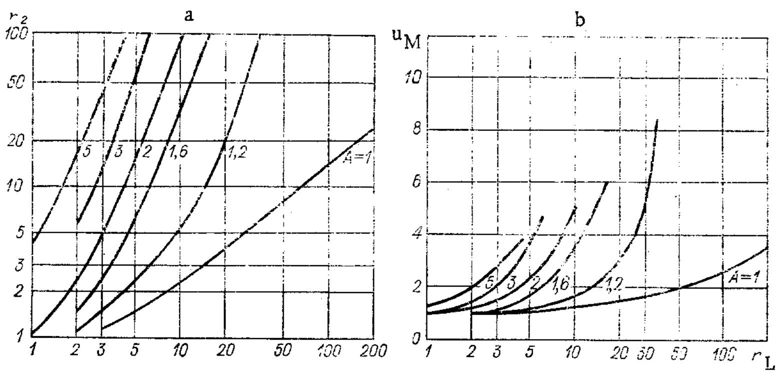

Fig. 4

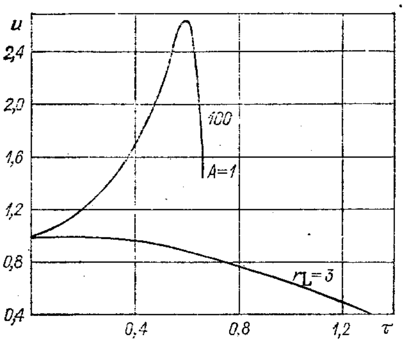

Fig. 5

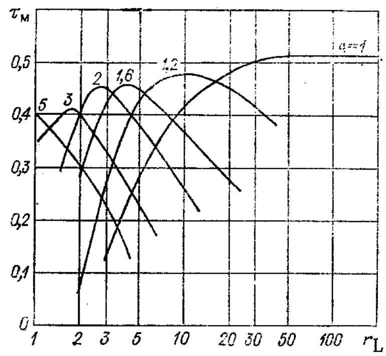

Fig. 6

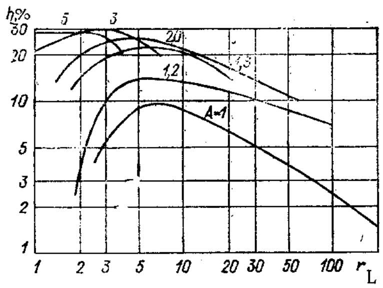

Fig. 7

ments on electrical explosion [3] that at the time of the so-called "current pause" the intensity of the current is in fact not zero, and, consequently, the resistance of an exploding wire is always finite. If one takes $r_2 = 100$ (see Fig. 4a), the corresponding values of the pulse amplitudes will not exceed 9 (Fig. 4b). The relationship $u(\tau)$ can be obtained in the following manner. One determines $r(\tau)$ and $r'(\tau)$ by numerical integration of Eq. (2.3) and then calculates $u(\tau) = (r'/r)/(r'/r)_{\tau=0}$. Using this method, voltage pulses were plotted for $A = 1$, $r_L = 3$, and $r_L = 100$ (Fig. 5). The time τ_M during which a pulse acts from its beginning until it reaches maximum value can be determined from Eq. (2.3),

$$\tau_M = \frac{1}{2r_L}\int_1^{r_2} \frac{(r + r_L)^2 dr}{r^3\left[\frac{r_L}{r} - \ln r + (A-1)r_L\right]}.$$

The integral is not expressed in elementary functions and was calculated on a computer (Fig. 6). It is clear from the curves that for a fixed load resistance r_L, an increase in the dimensionless energy A of the inductive storage can both lengthen and shorten the voltage pulse (i.e., both increase and decrease the time τ_M to reach the peak). Physically, this is explained by the competition of two factors; the power in the load grows as A increases but the total energy transferred also increases. The second factor dominates for small r_L and the first factor for large r_L. The total energy transferred to the load is defined as the storage energy after subtraction of the total energy absorbed by the disconnect,

$$Q_L = \frac{1}{2}LI_0^2 - m_0 q\left(1 - \frac{1}{r_1}\right).$$

Hence, the total efficiency is

$$\eta_t = 1 - \frac{1}{A}\left(1 - \frac{1}{r_1}\right).$$

It is clear that for $A \gg 1$ (i.e., for a disconnect with energy of the electrical explosion much less than the storage energy), η_t can be as close to one as desired. We note that this energy is transferred to the load in the asymptotic mode, i.e., in theory after an infinitely long time [in practice, after a time $(L/R_b)\tau_M + (2\text{-}3)(L/R_L)$]. There often arises a need to transfer a given energy in a limited time. There is therefore practical interest in the efficiency of a rapid transfer to the load in the time τ_M from the beginning of a pulse until the peak value of the energy

$$\frac{1}{2} L I_0^2 (1 - i_2^2) - m_0 q \left(1 - \frac{1}{r_2}\right),$$

where i_2 is the dimensionless value of the storage current at the time τ_M. Substituting

$$i_2 = u_M \frac{1 + r_L/r_2}{1 + r_L},$$

we find the efficiency for rapid transfer is

$$\eta = 1 - \frac{1}{2Ar_2}\left(1 + \frac{r_2}{r_L}\right)^2 - \frac{1}{A}\left(1 - \frac{1}{r^2}\right).$$

The dependence of η on A and r_L is given in Fig. 7 by curves from which it is clear that η cannot exceed 30%. It is interesting to note that the maximum efficiency in the transfer of energy from an inductive storage to an inductive load is 25% [4].

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